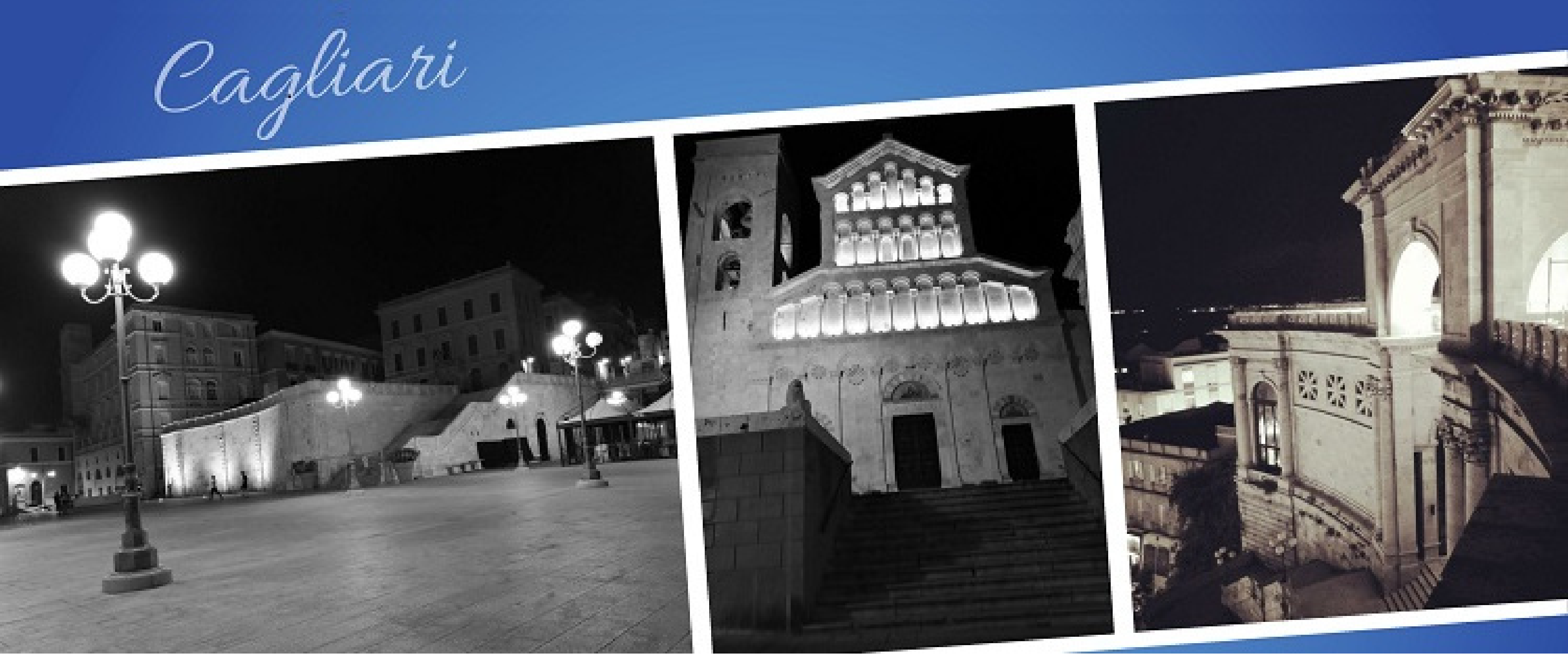
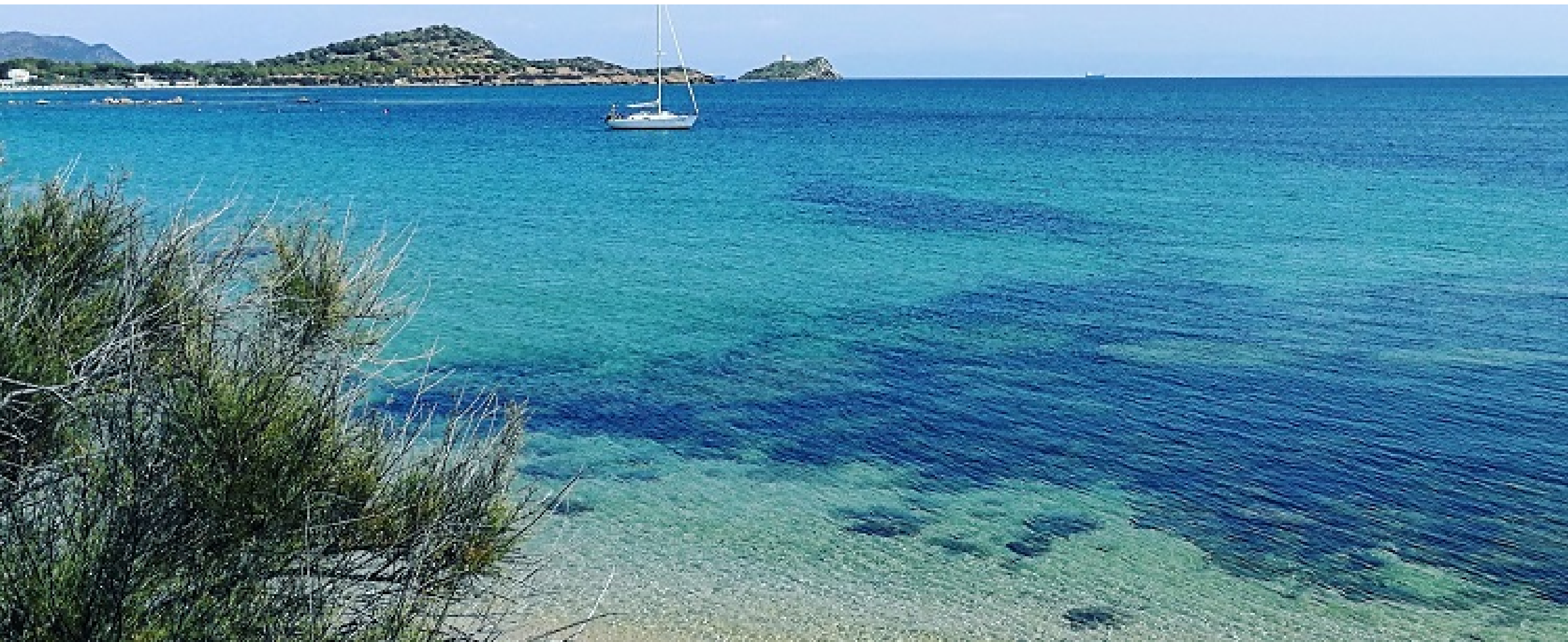


IWANASP18

International Workshop on Analysis and Numerical Approximation of Singular Problems

In memory of *Christopher Baker* and *Sebastiano Seatzu*

Cagliari, Italy
September 4-6, 2018



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Financial support is acknowledged from:

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Scientific Program

- **Invited Speakers**
  - **Peter Junghanns**, Technische Universität Chemnitz, Germany
  - **Giuseppe Mastroianni**, University of Basilicata, Italy
  - **Arvet Pedas**, University of Tartu, Estonia
  - **Jason Roberts**, University of Chester, United Kingdom
  - **Yuesheng Xu**, Old Dominion University in Norfolk, Virginia, United States
- **Contributed talks**
  - **Sílvia Barbeiro**, University of Coimbra, Portugal
  - **Rosanna Campagna**, University of Naples Federico II, Italy
  - **Elena Chistyakova**, Institute for System Dynamics and Control Theory SB RAS, Russia
  - **Maria Carmela De Bonis**, University of Basilicata, Italy
  - **Patricia Díaz de Alba**, University of Cagliari, Italy
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  - **Neville Ford**, University of Chester, UK
  - **Hanane Kaboul**, University of Biskra, Algeria
  - **Rekha Kulkarni**, I.I.T. Bombay, India
  - **Laurence Grammont**, Université Jean Monnet, Université de Lyon, France
  - **Concetta Laurita**, University of Basilicata, Italy
  - **Pedro Lima**, University of Lisbon, Portugal
  - **Incoronata Notarangelo**, University of Basilicata, Italy
  - **Donatella Occorsio**, University of Basilicata, Italy
  - **Maria Grazia Russo**, University of Basilicata, Italy
  - **Giada Serafini**, University of Basilicata, Italy
  - **Liubov Solovarova**, Matrosov Institute for System Dynamics and Control Theory of SB RAS, Russia
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Abstracts

Invited Speakers

FROM CLASSICAL APPROACHES TO C^*-ALGEBRA TECHNIQUES IN THE NUMERICAL ANALYSIS OF SINGULAR INTEGRAL EQUATIONS

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The application of Cauchy singular and hypersingular integral equations, for example in airfoil theory and elasticity theory, and the theory of their numerical solution have a long history. The so-called classical collocation method for equations of the type

$$a(x)u(x) + \frac{b(x)}{\pi}\int_{-1}^{1}\frac{u(y)\,dy}{y-x} + \int_{-1}^{1} h(x,y)u(y)\,dy = f(x)\,, \quad -1 < x < 1\,, \tag{1}$$

is based on formulas like

$$\frac{1}{\pi}\int_{-1}^{1}\frac{T_n(y)\,dy}{(y-x)\sqrt{1-y^2}} = U_{n-1}(x)\,, \quad -1 < x < 1\,,\ n = 0,1,2,\dots\,, \tag{2}$$

where $T_n(x)$ and $U_n(x)$ are the normalized Chebyshev polynomials of degree n and of first and second kind, respectively. Originally, this method was restricted to equations (1) with constant coefficients $a(x) \equiv a$ and $b(x) \equiv b$.

Basically, there exist two different ways to generalize the classical collocation method for equations with variable coefficients. The first one is by construction of generalized Jacobi polynomials satisfying a relation like (2) which is closely connected with the coefficients $a(x)$ and $b(x)$. The second one is still based on classical Chebyshev polynomials and their zeros independ from the coefficients in the equation (1). The numerical methods based on the first approach need much time for preprocessing, namely for the computation of the nodes and weights of generalized Jacobi polynomials. But the essential condition for their applicability is only the unique solvability of equation (1). The preprocessing for the methods based on the second approach is very cheap, but in general the invertibility of more than one operator is necessary and sufficient for their applicability. The investigation of the stability of such methods is based on the application of C^*-algebra techniques.

The talk gives an overview of these developments during the last 25 years and concentrates on recent results for equations of the form (1), where kernel functions $h(x,y) = k\left(\frac{1+x}{1+y}\right)\frac{1}{1+y}$ of Mellin type occur which are important in applications, for example in two-dimensional elasticity theory.

Polynomial approximation of functions with singular points

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The polynomial approximation of non-continuous or non-L^p-integrable functions occurs in several contexts. In many applications, functions may have a finite number of strong singularities at the endpoints of the interval of definition and/or at some inner points.

A frequently used procedure consists of multiplying the function f by a suitable weight u so that fu turns out to be continuous or belongs to L^p; then fu can be approximated by a sequence of the form $\{P_m u\}$, where P_m is a polynomial of degree m. The choice of the weight u is related to the "pathology" of the function f (see [2, 1]).

In this talk we are going to show the main results in the case of singularities at the endpoints of the interval and we will mention the case of inner singularities. The behaviour of some concrete approximation operators will be also illustrated.

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NUMERICAL SOLUTIONS AND THEIR SUPERCONVERGENCE FOR FRACTIONAL DIFFERENTIAL EQUATIONS

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We propose and analyze a numerical method for solving initial and boundary value problems for fractional differential equations with Caputo type fractional derivatives. Usually, we cannot expect the solutions of such equations to be smooth on the whole interval of integration, which is a challenge to the convergence analysis of numerical methods. Therefore, using an integral equation reformulation of the original problem, we first study the regularity of the exact solution. Based on the obtained smoothness properties and spline collocation techniques, the numerical solution of the problem is discussed. Optimal convergence estimates are derived and aspects related to the superconvergence of the proposed algorithms are presented. A numerical illustration is also given.

UNDERSTANDING THE INTERACTION BETWEEN FINANCIAL MARKETS WITH ECOLOGICAL MODELLING TECHNIQUES

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A number of recent global and continent-wide events and decisions have brought financial markets and how they interact under scrutiny. We compare the historical indices of two such financial markets, which (on a global level) can be considered relatively close geographically and could be considered two of the major markets (if not the major markets) within Europe: The London FTSE100 and the Frankfurt DAX. The purpose of our comparison is to see if, by treating these markets as different species operating in the same ecosystem some insight can be gained by modelling the two markets with ecological models of interacting species. In particular, we are looking to ascertain if, historically, we can identify periods of time where the markets are exhibiting behaviour which may, in ecological terms be described as mutualistic, competitive or predation by one species on the other. Furthermore, we ask the question as to whether or not such information, when coupled with our knowledge of corresponding economic events at the time can provide some indication as to future market behaviour and the types of interactions to expect at key times between the two markets under consideration.

In order to seek answers to these questions we introduce delay and feedback into a number of ecological models and consider the dynamical behaviour of such models, before considering other aspects such as parameter estimation and the sensitivity of the system to changes in those parameters.

FAST SOLUTIONS OF BOUNDARY INTEGRAL EQUATIONS FOR THE POISSON EQUATION

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We shall present recent development of fast solutions of boundary integral equations that are reformations of the Poisson equation. By employing hyperbolic cross approximations for the integral operators and the Newton potential, we develop a fast method for solving the equation. Optimal convergence and computational complexity for the proposed method will be presented and numerical results will be shown to verify the theoretical estimates.

Contributed Talks
Approximation Theory

FINITE ELEMENT ERROR ANALYSIS FOR PROBLEMS WITH LOW REGULAR SOLUTIONS

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We consider the numerical solution with finite element methods of elliptic boundary value problems with both inhomogeneous Dirichlet and Neumann boundary conditions. The focus of this talk is to derive L^2 error estimates without restrictive regularity assumptions on the solutions of the original and adjoint problems. We analyze in particular the effect of the choice of the discrete Dirichlet data on the estimates. To illustrate the theoretical results, some numerical examples will be presented.

ROBUST MESHFREE PDE SOLVER FOR SOURCE-TYPE FLOWS IN POROUS MEDIA

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Radial Basis Function (RBF)-based methods, taking advantage of being meshfree, are nowadays widely adopted tools for solving Partial Differential Equations (PDEs) via collocation schemes, see e.g. [1]. Generally, the local approximants and consequently also the global ones may suffer from instability due to the ill-conditioning of the interpolation matrices. To avoid this drawback, which becomes even more evident when approximationg functions with singularities or discontinuities, we suggest a methodology consisting in building the differentiation matrices via the so-called Variably Scaled Kernels (VSKs). VSK were first introduced in [2]. Furthermore, to manage the sparsity of the collocation systems we adopt the Partition of Unity Method (PUM), refer e.g. to [3]. In this framework, we propose an efficient and robust approach which turns out to be suitable in a *realistic* engineering problem where a steady state flow is assumed determined by a pulse-like extraction of water at a constant volumetric rate [4]. This leads to an elliptic PDE with a singular forcing term. Generally an adopted methodology is to use numerical schemes with fine grid level of discretizations near singularities. In this work, conversely, we propose to apply an emerging strategy to tackle such a type of flow configuration in porous formations.

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Some Properties of the Linear Differential Algebraic Equations Perturbed by the Fredholm Operators

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Systems of ordinary differential equations with a singular matrix multiplying the higher derivative of the desired vector-function are commonly referred to as differential algebraic equations (DAEs), and linear DAEs generally have the form

$$\Lambda_k x := \sum_{i=0}^{k} A_i(t)x^{(i)}(t) = f(t),\ t \in T := [\alpha,\ \beta], \tag{3}$$

where $A_i(t)$ are $n \times n$-matrices, $x(t)$ and $f(t)$ are the desired and the given vector-functions, correspondingly, $x^{(i)}(t) = (d/dt)^i x(t),\ x^{(0)}(t) = x(t)$, and

$$\det A_k(t) = 0\ \forall t \in T. \tag{4}$$

Usually, a set of the initial data is given

$$x^{(j)}(\alpha) = a_j,\ j = \overline{0, k-1}, \tag{5}$$

where a_j are vectors from $\mathbf{R}^n$. For $k = 1$ in (3), DAEs have been fairly well studied. In this talk we consider properties of DAEs perturbed by the Fredholm operator when $k > 1$:

$$(\Lambda_k + \lambda\Phi)x := \sum_{i=0}^{k} A_i(t)x^{(i)}(t) + \lambda \int\limits_{\alpha}^{\beta} \mathrm{K}(t,s)x(s)ds = f(t), \tag{6}$$

where $t \in T,\ \lambda$ is some parameter, $\mathrm{K}(t,s)$ is $n \times n$-matrix, with the initial data (5). We focus on the solvability conditions for the initial problem (6), (5) and propose a numerical algorithm of solution based on the least squares method.

This work has been partially supported by the Russian Foundation for Basic Research, Grants Nos. 18-51-54001, 18-01-00643.

A QUADRATURE METHOD FOR A SINGULAR INTEGRO-DIFFERENTIAL EQUATION IN WEIGHTED ZYGMUND SPACES WITH UNIFORM NORM

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This talk deals with the numerical solution of singular integro-differential equations of the following type

$$\sigma(x)u(x) + au'(y) + \frac{b}{\pi}\int_{-1}^{1}\frac{u'(x)}{x-y}dx + \frac{1}{\pi}\int_{-1}^{1}k(x,y)u(x)dx = g(y), \quad |y| \le 1,$$

where the unknown solution u satisfies the additional conditions $u(-1) = u(1) = 0$, $a, b \in \mathbf{R}$ are known and σ, k, g are given functions.
Several authors have studied this type of integro-differential equations and related numerical methods (see, for example, [1], [2], [3, Section 3]).
We propose a numerical method of quadrature type and we prove that it is stable and convergent giving error estimates in weighted spaces of continuous functions equipped with uniform norm. Moreover we show some numerical tests that confirm the theoretical estimates.

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Recovering the electrical conductivity of the soil via a linear integral model

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This work investigates a linear model that involves Fredholm integral equations of the first kind defined on the positive semiaxes used to describe the interaction of an electromagnetic field with the soil [3]. The aim is to detect or infer, by non destructive investigation of soil properties, inhomogeneities in the ground as well as the presence of particular conductive substances.

To find the solution of the problem, we propose some numerical methods based on splines and Bernstein polynomials, combined with a suitable regularization technique as the Truncated (Generalized) Singular Value Decomposition [1, 2].

Finally, we test the effectiveness of the different approaches on synthetic data sets.

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SEBASTIANO SEATZU'S CONTRIBUTION TO THE NUMERICAL TREATMENT OF NONLINEAR EVOLUTION EQUATIONS

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Sebastiano Seatzu has been full professor of numerical analysis at the University of Cagliari, Italy, since 1980. He coauthored 3 books and 86 papers, mostly published in international journals, and he was coeditor of 3 books. Most of his publications were in numerical analysis, but he also coauthored several papers in physics and chemical physics.

In the past 15 years, he worked mainly on the numerical solution of integral equations of the first kind, numerical linear algebra of structured matrices, numerical treatment of ill-conditioned systems of linear equations, numerical techniques for integral equations with structured kernels, numerical solution of nonlinear evolution equations, and analytical and numerical methods related to the design of photonic crystals.

I was co-author, along with Cornelis van der Mee, of his last six papers. They are related to nonlinear partial differential equations (NPDE) of integrable type, which have important physical applications. Indeed, they are used to describe electromagnetic waves in optical fibers, surface wave dynamics, charge density waves, breaking wave dynamics, etc.

In this talk, we focus on the research that was currently in progress when he left us on February 13th, 2018, namely, the numerical treatment of the Korteweg-de Vries (KdV) equation, which governs the propagation of surface water waves in long, narrow, shallow canals [1]

$$\begin{cases} \dfrac{\partial q(t,x)}{\partial t} - 6q(t,x)\dfrac{\partial q(t,x)}{\partial x} + \dfrac{\partial^3 q(t,x)}{\partial x^3} = 0, \quad x \in \mathbb{R},\ t \in \mathbb{R}^+, \\ q(0,x) = q(x). \end{cases}$$

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Christopher T H Baker (1939-2017): His contribution to the field

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Christopher Baker was one of the first numerical analysts to investigate integral equations, including those involving a singularity. He was the author of several key books on the subject and was influential in shaping theoretical and numerical work on integral and integro-differential equations, retarded functional differential equations and stochastic differential equations. More recently, he developed a keen interest in modelling (problems 'with memory and after-effect') particularly in the biosciences and immunology. His focus was always on understanding both the nature of the problem under consideration, and the useful questions to answer for applications purposes.

In this talk, by his former PhD student and collaborator of more than 30 years, we provide a brief overview of his main contribution and then focus on some of his most recent (and previously unpublished) results.

The product integration method for a weakly singular Hammerstein equation

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This talk deals with nonlinear Fredholm integral equations "Hammerstein equation" of the second kind. We study the case of a weakly singular kernel and we set the problem in the space the space of integrable functions over closed interval in $\mathbb{R}$, $L^1([a,b],\mathbb{C})$. We extend the product integration scheme from $C^0([a,b],\mathbb{C})$ to $L^1([a,b],\mathbb{C})$.

Discrete modified projection method for nonlinear integral equations with non-smooth kernels

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Consider a nonlinear integral equation $x - K(x) = f$, where K is a Urysohn integral operator with a Green's function type kernel. Approximate solutions using the Galerkin and the iterated Galerkin method based on the orthogonal projection onto a space of discontinuous piecewise polynomials are investigated in [1]. Orders of convergence of the approximate solution using the iterated modified projection method are obtained in [2]. In this paper we consider the discrete versions of these methods and specify a choice of numerical quadrature which preserves the orders of convergence. Numerical results are given to validate the theoretical results.

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A PROJECTION BASED REGULARIZED APPROXIMATION METHOD FOR ILL-POSED OPERATOR EQUATIONS

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Problem of solving Fredholm integral equations of the first kind is a prototype of an ill-posed problem of the form $T(x) = y$, where T is a compact operator between Hilbert spaces. Regularizations and discretizations of such equations are necessary for obtaining stable approximate solutions for such problems. For ill-posed integral equations, a quadrature based collocation method has been considered by Nair (2012) for obtaining discrete regularized approximations. As a generalization of that, a projection collocation method has been studied in 2016. In both of the considered methods, the operator T is approximate by a sequence of finite rank operators. In the present paper, the authors choose to approximate TT^* by finite rank operators. It is found that in some cases, the derived estimates are improvements over the previous estmiates.

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A BIEM FOR MIXED BOUNDARY VALUE PROBLEMS ON NONSMOOTH BOUNDARIES

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In this talk we propose a new approach to the numerical solution of the mixed Dirichlet-Neumann boundary value problem for the Laplace equation in planar domains with piecewise smooth boundaries.

Using the single layer representation of the potential and employing the Dirichlet and Neumann boundary conditions, the differential problem is reformulated in the form of a system of boundary integral equations (BIE), whose unknown is the single layer density function on the boundary.

Then, we consider an associated perturbed BIE system and present a Nyström-type method for its numerical solution.

As Mellin type integral operators are involved, we need to modify the method close to the corners in order to prove its stability and convergence.

Some numerical tests are given showing the efficiency of the proposed method.

Numerical Solution of a Volterra Integral Equation of the Third Kind

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In this work, an operational matrix of fractional integration based on an adjustment of hat functions is used for solving a class of third-kind Volterra integral equations with weakly singylar kernel. We show that the application of this numerical technique reduces the problem to a linear system of equations that can be efficiently solved. Some numerical examples are considered to demonstrate the accuracy and efficiency of the proposed method.

ON PRODUCT INTEGRATION METHODS FOR INTEGRAL EQUATIONS WITH WEAKLY SINGULAR KERNELS

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Let us consider integral equations of the form

$$f(x) - \int_I k(x,y) f(y) w(y)\,\mathrm{d}y = g(x)\,, \qquad x \in I\,,$$

where k and g are given functions, f is the unknown functions, I is a bounded or unbounded interval and w is a nonstandard weight function on I, for instance

$$w(x) = \mathrm{e}^{-1(1-x^2)^\alpha}\,, \quad \alpha > 0\,, \quad x \in I = (-1,1)$$

or

$$w(x) = x^\alpha \mathrm{e}^{-x^\beta}\,, \quad \alpha > -1\,,\, \beta > \frac{1}{2}\,, \quad x \in I = (0,+\infty)\,.$$

This talk is devoted to the theoretical investigation of the Nyström methods based on a suitable product quadrature rule in the case of a weakly singular kernel k and a locally smooth g. In fact, if k has weak inner singularities along a line, the method based on the Gaussian rule related to w cannot be used. So, replacing f by a suitable Lagrange polynomial we obtain a sequence of operators $\{K_m\}_m$. We prove that this sequence converges to the integral operator K and is collectively compact.

References

[1] P. Junghanns, G. Mastroianni and I. Notarangelo, *On Nyström and product integration methods for Fredholm integral equations*, In: Contemporary Computational Mathematics - a celebration of the 80th birthday of Ian Sloan (J. Dick, F.Y. Kuo, H. Woźniakowski, eds.), Springer International Publishing, 2018, pp. 645 – 673. doi: 10.1007/978-3-319-72456-0_29

EXTENDED PRODUCT INTEGRATION RULES IN $[-1,1]$

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In this talk we will consider suitable product integration rules for integrals of the type

$$I(f,y) := \int_{-1}^{1} f(x)K(x,y)w(x)dx,$$

where $w(x) = (1-x)^{\alpha}(1+x)^{\beta}$ is a Jacobi weight, K is a given kernel presenting a pathological behaviour (for instance high oscillations or weak singularities). Denoting by $\{p_m(w)\}_m$ the sequence of the orthonormal polynomials w.r.t. w, let $L_{m,m+1}(w,w,f)$ be the extended Lagrange polynomial interpolating f at the zeros of $Q_{2m+1} = p_{m+1}(w)p_m(w)$ and let $\Sigma^*_{m,m}(f,y)$ be the extended product integration rule obtained by approximating f with $L_{m,m+1}(w,w,f)$, i. e.

$$\begin{aligned} I(f,y) &= \int_{-1}^{1} L_{m,m+1}(w,w,f)K(x,y)w(x)dx + e_m(f,y), \\ &:= \Sigma^*_{m,m}(f,y) + e_m(f,y). \end{aligned}$$

Denoting by $\Sigma_m(f,y)$ the usual product integration rule based on the zeros of $p_m(w)$, i.e. $\Sigma_m(f,y) := \int_{-1}^{1} L_m(w,f,x)K(x,y)w(x)dx$ and assuming that both the sequences $\{\Sigma_m(f)\}_m$ and $\{\Sigma^*_{m,m}(f)\}_m$ approximate the integral with the same rate of convergence, it makes sense to consider the mixed sequence $\{\Sigma_m(f), \Sigma^*_{m,m}(f)\}_m$ rather than the usual $\{\Sigma_m(f)\}_m$. In this way we can double the number of nodes of the quadrature formula, by reusing m samples of the function f. This approach is especially relevant when m is "large" and the procedure for computing the zeros and the coefficients of the quadrature rule can fail. Moreover, also the coefficients of the extended rule $\Sigma^*_{m,m}(f)$ can be computed through those of $\Sigma_m(f)$. We will show the stability and convergence of the mixed scheme, giving also some numerical tests, which confirm the theoretical estimates.

A GLOBAL APPROXIMATION METHOD FOR MELLIN SINGULAR INTEGRAL EQUATIONS

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Mellin singular integral equations arise in several applications. For instance they occur in crack problems in linear elasticity, or in the so called planar radiosity equation, relating the radiosity at points of a surface to the reflectivity and the emissivity at such points (see for instance [1, 2]).

The Mellin singular integral equations have the following form:

$$f(y) + \int_{-1}^{1} K(x,y) f(x) dx + \int_{-1}^{1} H(x,y) f(x) dx = g(y)$$

where $K(x,y) = \pm \frac{k\left(\frac{1+y}{1+x}\right)}{1+x}$, s.t. $\int_0^\infty \frac{k(x)}{x} dx < \infty$, and $H(x,y)$, g are known continuous functions in $[-1,1]^2$ and $[-1,1]$, respectively, while f is the unknown solution. The main difficulty in treating this kind of singular equation is that the Mellin operator $\int -1^1 K(x,y) f(x) dx$ is not compact.

The proposed method consists in a discrete collocation method based on the Lagrange interpolation. The interpolation process is constructed on the zeros of the Legendre polynomials and on the additional knots ± 1. Moreover the integrals involved in the construction of the matrix of coefficients of the linear system of the method are approximated by means of the standard Gauss-Legendre rule, if the collocation point is far from -1, and with a dilation tecnique if the collocation point is close to -1.

References

[1] K. Atkinson, *The planar radiosity equation and its numerical solution*, IMA Journal of Numerical Analysis 20 (2000), pp. 303–332.

[2] G. Monegato, S. Prössdorf, *On the numerical treatment of an integral equation arising from a cruciform crack problem*, Math. Methods Appl. Sci. 12 (1990), pp. 489—502.

NUMERICAL TREATMENT FOR BISINGULAR CAUCHY INTEGRAL EQUATIONS

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In this talk we investigate the numerical treatment of the bisingular integral equation of the first kind, defined on the square $S = [-1,1] \times [-1,1]$, having the following form

$$(D+K)f = g$$

where f is the bivariate unknown function, g is a given right-hand side, D is the dominant operator

$$Df(t,s) = \frac{1}{\pi^2} \oint_S \frac{f(x,y)}{(x-t)(y-s)} \sqrt{\frac{1-x}{1+x}} \sqrt{\frac{1-y}{1+y}} dxdy$$

and K is the perturbation operator

$$Kf(t,s) = \int_S f(x,y) k(x,y,t,s) \sqrt{\frac{1-x}{1+x}} \sqrt{\frac{1-y}{1+y}} dxdy$$

with k a given kernel function.

We propose two different methods: the first is a direct method, the second an indirect one. In both cases, we examine the stability, discuss the convergence and analyze the conditioning of the involved linear systems. Moreover, some numerical tests, which confirm the theoretical estimates, are proposed.

NUMERICAL SOLUTION OF DIFFERENTIAL-ALGEBRAIC EQUATIONS WRITTEN IN INTEGRAL FORM

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Consider

$$q(t)A(t)x^{'}(t)+B(t)x(t)=f(t),\ x(0)=x_0,\ t\in[0,T], \tag{7}$$

where $q(t)\equiv 1$ or $q(t)=t^{\alpha}$, $0<\alpha<1$, $A(t)$, $B(t)$ are $(n\times n)$-matrices, $f(t)$ and $x(t)$ are the given and unknown n-dimensional vector-functions, respectively. It is assumed that $detA\equiv 0$ and the initial condition are consistent with the right-hand part. Systems (7) are called differential-algebraic equations (DAEs). If $q(t)=t^{\alpha}$, then such problems are called DAEs with a weakly singular point.

We propose to rewrite (7) as

$$A(t)x(t)+\int_0^t(q^{-1}(\tau)B(\tau)-A^{'}(\tau))x(\tau)d\tau= \tag{8}$$

$$=\int_0^t q^{-1}(\tau)f(\tau)d\tau+A(0)x(0).$$

Special algorithms are proposed for numerical solution of problem (8). Advantages of these methods are discussed.

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APPROXIMATING THE SOLUTION OF INTEGRO-DIFFERENTIAL PROBLEMS VIA THE SPECTRAL TAU METHOD

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The Lanczos' Tau method is examined in detail from a variety of aspects to provide a stable implementation for its operational version. We concentrate on avoiding basis transformation, on performing polynomial evaluations directly on the orthogonal basis, on tackling nonlinear problems and how to effectively compute polynomial approximations from non-polynomial coefficient functions. The ultimate goal is to deploy a robust and efficient numerical library, the `Tau Toolbox`, able to deliver approximate solutions of integro-differential problems.

References

[1] C. Lanczos, *Trigonometric interpolation of empirical and analytical functions*, Journal of Mathematics and Physics, 17-1 (1938), pp. 123–199.

[2] P. B. Vasconcelos, J. Matos, and M. Trindade, *Spectral Lanczos Tau method for systems of nonlinear integro-differential equations*, Integral Methods in Science and Engineering, Volume 1: Theoretical Techniques, (2017), pp. 305–314.

[3] M. Trindade, J. Matos and P. B. Vasconcelos, *Dealing with Functional Coefficients Within Tau Method*, Mathematics in Computer Science, (2018) pp. 1–13.

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10:25 - 10:50	Contributed Talk in memory of Sabastiano Seatzu	Contributed talk	Contributed talk
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