Let \( w(x) = e^{-x^\beta}x^\alpha, \alpha > -1, \beta > \frac{1}{2} \) be a Generalized Laguerre weight, and denote by \( \{p_m(w)\} \) the corresponding sequence of orthonormal polynomials. Setting \( \bar{w}(x) = xw(x) \), let \( \{p_m(\bar{w})\} \) the sequence of orthonormal polynomials corresponding to \( \bar{w} \). We prove that the polynomial \( Q_{2m+1} = p_{m+1}(w)p_m(\bar{w}) \) has simple zeros and that they are also well distributed in some sense.

In view of this property we propose two different applications: the extended interpolation polynomial \( L_{2m+2}(w, \bar{w}, f) \), defined as the Lagrange polynomial interpolating a given function \( f \) at the zeros of \( Q_{2m+1} \) and on additional knots, estimating the Lebesgue constants in some weighted spaces. Moreover, we propose a method to approximate the Hilbert transform on the real positive semiaxis by a suitable Lagrange interpolating polynomial.