Iterative regularization for nonlinear imaging in Banach spaces

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Several regularization methods for inverse problems have been firstly and deeply analyzed in the context of the Hilbert space L^2. Unfortunately, regularization methods in Hilbert spaces usually show over-smoothness, which is a drawback in all the practical imaging applications, where the true solutions have natural discontinuities.

More recently, some regularization methods have been introduced and investigated in the context of more general Banach spaces. Due to the geometrical properties of Banach spaces, these new regularization methods allow us to obtain solutions endowed with lower over-smoothness, which results, as instance, in a better localization and restoration of the edges in image deblurring.

In this talk, we consider the nonlinear operator equation $F(x) = y$, where $F : X \longrightarrow Y$ is a nonlinear and ill-posed operator between the two Banach spaces X, Y, and $x \in X$ is the "cause" to be found of some known "effects" $y \in Y$. In particular, we analyze an iterative method for the minimization of the functional $\Phi(x) = \frac{1}{p}||F(x) - y||_Y^p$, where $Y = L^p$ is the Banach space of p-th power Lebesgue integrable functions, with $1 < p < +\infty$.

The proposed iterative algorithm in the framework of L^p is a nonlinear generalization of the simple Landweber method for nonlinear equations in L^2. The algorithm is applied to a nonlinear inverse scattering problem where the dielectric distributions x (i.e., the image to restore) of a $2D$ domain have to be recovered by means of its scattered microwave field y (i.e., the known data) outside the domain. We will show how the new computational results in Banach spaces well outperform classical "Hilbertian regularization".