

QUADRATURE ERRORS, DISCREPANCIES AND VARIATIONAL DITHERING

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The stippling technique places black dots such that their density gives the impression of tone. This is the first paper that relates the distribution of stippling dots to the classical mathematical question of finding 'optimal' nodes for quadrature rules. More precisely, we consider quadrature error functionals on reproducing kernel Hilbert spaces (RKHSs) with respect to the quadrature nodes and suggest to use optimal distributions of these nodes as stippling dot positions. Interestingly, in special cases, our quadrature errors coincide with discrepancy functionals and with recently proposed attraction-repulsion functionals. Our framework enables us to consider point distributions not only in R^2 but also on the torus T^2 and the sphere S^2 . For a large number of dots the computation of their distribution is a serious challenge and requires fast algorithms. To this end, we work in RKHSs of bandlimited functions, where the quadrature error can be replaced by a least squares functional. We apply a nonlinear conjugate gradient (CG) method on manifolds to compute a minimizer of this functional and show that each step can be efficiently realized by nonequispaced fast Fourier transforms. We present numerical stippling results on S^2 .

This is joint work with M. Gräf and D. Potts, University of Chemnitz, Germany.