Deflation based preconditioning of linear systems of equations

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For most real-world problems Krylov space solvers only converge in a reasonable number of iterations if a suitable preconditioning technique is applied. This is particularly true for problems where the linear operator has eigenvalues of small absolute value — a situation that is very common in practice. One suitable technique for dealing with such problems is to identify an approximately invariant subspace $\mathcal{Z}$ that belongs to the set of these small eigenvalues. By using an orthogonal projection along $\mathcal{Z}$ the Krylov solver can then be applied only to the orthogonal complement by restricting the operator accordingly. The basis constructed implicitly or explicitly by this restricted operator should then be augmented by a set of basis vectors for $\mathcal{Z}$. There are various ways to handle and implement this approach. They differ not only algorithmically and numerically, but sometimes also mathematically. Some keywords associated with such methods are '(spectral) deflation', 'augmented basis', 'recycling Krylov subspaces', and 'singular preconditioning'.

While we quickly also review the 'symmetric case', where the linear system is Hermitian (or real and symmetric), we are mostly interested in the 'nonsymmetric case', where our main message is that the orthogonal projection should be replaced by a suitable oblique projection, so that when its nullspace is invariant, so is its range. For details, see [1].

References

[1] Martin H. Gutknecht, *Spectral Deflation in Krylov Solvers: a Theory of Coordinate Space Based Methods*, submitted (2011).