Matching moments and Krylov subspace methods

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Matching moments is inherently linked with numerical quadrature, continued fractions and orthogonal polynomials. The concept of moments arose with the work of Chebyshev, Markov and Stieltjes in the second half of the 19th century; the related numerical quadrature with Gauss in 1814, with further founding contributions due to Jacobi, Christoffel, Markov, Stieltjes and many others. The related fundamental concept of continued fractions can essentially be rooted back to Euclid and other ancient mathematicians; see the thorough descriptions given by Brezinski in several books, papers and essays. Stieltjes published in 1894 his analytic theory of continued fractions with an impact in forming foundations of functional analysis by Hilbert in 1906 - 1912, as well as in forming mathematical foundations of quantum mechanics by von Neumann in 1927 - 1932.

The original work of Krylov from 1931 refers to the work of Jacobi from 1846. Its algebraic formulation, with using what we now call the Krylov sequence, was given by Gantmacher in 1934. In modern computational mathematics, sciences and engineering, many ideas behind Krylov subspace methods and matching moments model reduction (in approximation of large scale dynamical systems and elsewhere) resemble the classical concepts mentioned above. Surprisingly, several important works which made these links transparent remained almost unknown; see, in particular, the work of Vorobyev from 1958 which would be without its popularization by Brezinski essentially forgotten.

In agreement with the views presented by Brezinski, Golub and Meurant, we consider viewing relevant matrix computations as matching moments inspirational and useful. We will demonstrate this on several examples. In particular, we will address the question of cost evaluation and numerical stability in Krylov subspace iterations.