Preconditioner updates for sequences of symmetric positive definite linear systems arising in optimization

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We consider sequences of large and sparse linear systems of the form

$$(A + D_j)x_j = b_j, \qquad j = 1, \ldots, m,$$

where A is a symmetric positive definite matrix and D_j are positive definite diagonal matrices. The solution of such sequences often arises in optimization, e.g. in trust-region and regularization subproblems, in Levenberg-Marquadt approaches, in affine scaling methods for quadratic programming, and in nonlinear least-squares. Our interest is in the case where the systems are solved by using Krylov methods with preconditioning techniques.

The spectral properties of the matrices of the sequence may considerably differ. Therefore, it may be inappropriate to use a frozen preconditioner for all the systems. We are interested in forming an efficient preconditioner for each system of the sequence without recomputing the preconditioner from scratch, in order to reduce the overall computational cost.

In this talk, we discuss techniques to update an incomplete LDL^T factorization of the matrix A. The proposed procedures, extending previous work on shifted systems [1], are cheap and easy to implement. A theoretical justification of our approach is presented along with numerical experiments illustrating its performance.

References

[1] S. Bellavia, V. De Simone, D. di Serafino, B. Morini, *Efficient Preconditioner Updates for Shifted Linear Systems*, submitted.