Quadrature on the positive real line with quasi and pseudo orthogonal rational functions

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We consider a positive measure on $[0,\infty)$ and a sequence of nested spaces $\mathcal{L}_0 \subset \mathcal{L}_1 \subset \mathcal{L}_1 \cdots$ or rational functions with prescribed poles in $[-\infty, 0]$. Let $\varphi_k \in \mathcal{L}_k$ be the associated sequence of orthogonal rational functions. The zeros of φ_n can be used as the nodes of a rational Gauss quadrature formula that is exact for all functions in $\mathcal{L}_n \cdot \mathcal{L}_{n-1}$, a space of dimension $2n$. Quasi- and pseudo-orthogonal functions are functions in $\mathcal{L}_n$ that are orthogonal to some subspace of $\mathcal{L}_{n-1}$. Both of them are generated from φ_n and φ_{n-1} and depend on a real parameter τ. Their zeros can be used as the nodes of a rational Gauss-Radau quadrature formula where one node is fixed in advance and the others are chosen to maximize the subspace of $\mathcal{L}_n \cdot \mathcal{L}_{n-1}$ where the quadrature is exact. The parameter τ is used to fix a node at a pre-assigned point. The space where the quadratures are exact have dimension $2n-1$ in both cases but it is in $\mathcal{L}_{n-1} \cdot \mathcal{L}_{n-1}$ in the quasi-orthogonal case and it is in $\mathcal{L}_n \cdot \mathcal{L}_{n-2}$ in the pseudo-orthogonal case. Although the quasi and the pseudo orthogonal rational functions and their zeros have very similar properties theoretically, the pseudo orthogonal rational functions are computationally much less favorable as compared to the quasi orthogonal rational functions if we want to compute the nodes and weights via a generalized eigenvalue problem.