From optimal cubature formulae to Chebyshev lattices: a way towards generalised Clenshaw-Curtis quadrature

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A landmark paper for cubature formulae in two dimensions is [1]. It used the ideal-theoretical approach to construct cubature formulae of arbitrary degrees attaining Möller's lower bound and pointed the direction towards a multivariate extension of Clenshaw-Curtis quadrature. To develop this idea, we introduced Chebyshev lattices [2]. This is a framework for cubature with the Chebyshev weight function. In combination with hyperinterpolation theory, this can be used to construct multivariate Chebyshev approximations and interpolating cubature rules. These rules extends the idea of Clenshaw-Curtis quadrature, including the efficient implementations that use the fast Fourier transform (FFT), to higher dimensions. Our framework includes Morrow-Patterson rules as well as other (near-)optimal point sets in two dimensions (such as Padua points). Higher dimensional point sets due to Noskov and Godzina also fit into this framework.

References

[1] C. R. Morrow and T. N. L. Patterson. Construction of algebraic cubature rules using polynomial ideal theory. *SIAM Journal on Numerical Analysis*, 15(5):953–976, 1978.

[2] Koen Poppe and Ronald Cools. Chebyshev lattices, a unifying framework for cubature with the Chebyshev weight function. *BIT Numerical Mathematics*, 2011. Available online: DOI:10.1007/s10543-010-0300-6.