Order bases: computation and uses in Computer Algebra

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Let $\mathbf{F} \in \mathbf{K}[[x]]^{m \times n}$ be a matrix of power series over a field $\mathbf{K}$. Then a vector $\mathbf{p} \in \mathbf{K}[x]^{n \times 1}$ of polynomials gives an *order* σ approximation of $\mathbf{F}$, if

$$\mathbf{F} \cdot \mathbf{p} \equiv \mathbf{0} \bmod x^{\sigma},$$

that is, the first σ terms of $\mathbf{F} \cdot \mathbf{p}$ are zero. Examples of such problems include Padé, Hermite-Padé, Simultaneous-Padé approximants and their vector and matrix generalizations. The set of all such order $(\mathbf{F}, \sigma)$ approximations forms a module over $\mathbf{K}[x]$. An *order basis* - or minimal approximant basis or σ-basis - is a basis of this module having a type of minimal degree property.

In this talk we will describe how to efficiently compute order bases in exact arithmetic environments. This includes the case where coefficient growth is an issue (and so bit complexity is needed) along with the case when one uses only the complexity of the arithmetic operations. Finally, we describe the use of order bases in the area of computer algebra. This includes normal form computation for matrix polynomials and fast polynomial matrix arithmetic.