Smoothing the Gibbs phenomenon using Padé-Hermite approximants

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The aim of this talk is to propose a method to reduce the Gibbs phenomenon exhibited by the partial Fourier sums of a periodic function $f$, defined on $[-\pi, \pi]$, discontinuous at 0. Let $g_2$ denote the series such that $f(t) = \Re(g_2(e^{it}))$. Then, the goal is to approach $g_2$ on the unit circle (and more precisely its real part). It is typical that the singularity of the function $f$, located at 0 say, corresponds to a logarithmic singularity for $g_2$, then located at 1, and that this function $g_2$ is analytic in the complex plane, with a branch cut that can be taken as the interval $[1, \infty)$. Defining $g_1(z) = \log(1-z)$, we may consider the problem of determining polynomials $p_0, p_1, p_2$ such that

$$p_0(z) + p_1(z)g_1(z) + p_2(z)g_2(z) = O(z^{n_0+n_1+n_2+2}) \quad (z \to 0)$$

where $n_j$ denotes the degree of $p_j$, $j = 0, 1, 2$. We can then propose the Hermite-Padé approximant

$$\Pi_n(z) = -\frac{p_0(z) + p_1(z)g_1(z)}{p_2(z)}, \quad (1)$$

to approximate $g_2$.

We obtain rates of convergence of sequences of Hermite–Padé approximants for a class of functions known as Nikishin systems. Our theoretical findings and numerical experiments confirm that particular sequences of Hermite-Padé approximants (diagonal and row sequences, as well as linear HP approximants) are more efficient than the more elementary Padé approximants, particularly around the discontinuity of the goal function $f$. 