Radially orthogonal multivariate basis functions

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It is well-known that radial basis functions provide a practical way to interpolate large scattered datasets. It is equally well-known that the interpolation matrix may be singular or ill-conditioned for some of the basis functions. We establish a connection with spherical orthogonal polynomials $V_m(z), X = (x_1, \ldots, x_d) = (\xi_1 z, \ldots, \xi_d z), ||(\xi_1, \ldots, \xi_d)||_p = 1$, defined on the unit hyperball (in different norms) by

$$\int \cdots \int_{||X||_p \leq 1} w(|z|) \left(\sum_{k=1}^{d} x_k \xi_k \right)^i V_m \left(\sum_{k=1}^{d} x_k \xi_k \right) \, dX = 0, \; i = 0, \ldots, m-1.$$

Because of the orthogonality of these multivariate basis functions, the interpolation matrix is better conditioned. Also small Lebesgue constants are obtained. We show how the multivariate spherical orthogonal polynomials can be used in:

- collocation methods, to compute a multivariate analytic model representing the European call option price (and its Greeks) as obtained from the Black-Scholes differential equation,
- CAGD, where we show that a fully orthogonal multivariate basis set can be obtained with orthogonality between different basis functions of the same total degree which, in a multivariate setting, is termed mutual orthogonality,
- data fitting, illustrating the radial usage of the orthogonal basis functions, where the variable is the signed distance function, versus the cartesian usage, where the variable is a linear combination of the cartesian coordinates.