POD FOR COMPLEX BLACK-SCHOLES EQUATIONS

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In financial engineering in banks at least the Black-Scholes equations are solved very many times each day and so any numerical speed-up is welcome. To compute several financial options with local volatilities or/and jumps, we introduce a one dimensional Galerkin basis for solving the parabolic partial integro-differential equations which arise from an Itô calculus when the random evolution of the underlying asset is driven by a Wiener process, or a Lévy process or more generally, a time-inhomogeneous jump-diffusion process.

The choice of the basis of functions is driven by the 3 main constraints: the numerical efficiency in the computation of the basis, the suitable global shape so as to be a complete basis with correct asymptotic behavior at infinity and the capacity to compute all the correlation matrices with analytical formulas. Elementary solutions of the Black-Scholes equation with constant volatilities fit these 3 criteria.

A convergence proof is given and numerical tests are performed on calls with non constant volatilities such as CEV and Lévy processes with Merton's kernel because analytical solutions are known for these. The basis is tried also for calibration of a local volatilities.

The method is a Proper Orthogonalization Decomposition very similar to those used for the heat equations; however here the basis is known in closed form.

The method is very fast but hampered by the bad condition numbers of the linear systems and so it is limited to no more than 40 basis beyond which the systems are numerically unsolvable.

It can be extended to two dimensional problems such as basket option and stochastic volatility models and we will show good numerical results but again unless a better numerical solver is found - and perhaps it will have been found by October 2011 - the method is limited by the size of the matrices that the SVD solvers can handle.