

# THE EARLY HISTORY OF CONVERGENCE ACCELERATION METHODS

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We consider the history of convergence acceleration methods before 1723. Taken up methods are as follows. Names in brackets are discoverers or provers.

1. Using the correction terms [Mādhava (c.1400), Isaac Newton (1676)]
2. Linear acceleration
  - (i) the arithmetic weighted mean [Willebrord Snell (1621), Christiaan Huygens (1654)]
  - (ii) the Richardson extrapolation [C. Huygens (1654)]
  - (iii) the Euler transformation [I. Newton (1684), Jean Christophe Fatio de Duillier (before 1704)]
  - (iv) the iterated Richardson extrapolation [Katahiro Takebe (before 1711)]
3. Nonlinear acceleration
  - (i) the harmonic weighted mean [Nikolaus van Kues (before 1464), W. Snell (1621), C. Huygens (1654)]
  - (ii) the Aitken  $\Delta^2$  process [Takakazu Seki (1680)]

We give an asymptotic error estimate for each taken up method and focus on the Aitken  $\Delta^2$  process by Takakazu Seki and the iterated Richardson extrapolation by Katahiro Takebe.