# Relaxed mixed constraint preconditioners for generalized saddle point linear systems

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The solution of the (generalized) saddle point linear system of the form $\mathcal{A}\boldsymbol{x} = \boldsymbol{b}$, where $\mathcal{A} = \begin{bmatrix} A & B^\top \\ B & -C \end{bmatrix}$ and $A$ is symmetric positive definite, $C$ is symmetric semi-positive definite, and $B$ a full-rank rectangular matrix, is encountered in many field such as e.g. constrained optimization, least squares, coupled consolidation problems and Navier-Stokes equations. Iterative solution is recommended against direct factorization methods due to the extremely large size of these systems. We propose here a development of the Mixed Constraint Preconditioners (MCP) introduced in [1] which is based on two preconditioners for $A$ ($P_A$ and $\widetilde{P_A}$) and a preconditioner ($P_S$) for a suitable Schur complement matrix $S = B\widetilde{P_A}^{-1}B^\top + C$. The family of Relaxed MCP is denoted by $\mathcal{M}^{-1}(\omega)$ where

$$\mathcal{M}(\omega) = \begin{bmatrix} I & 0 \\ BP_A^{-1} & I \end{bmatrix} \begin{bmatrix} P_A & 0 \\ 0 & -\omega P_S \end{bmatrix} \begin{bmatrix} I & P_A^{-1}B^\top \\ 0 & I \end{bmatrix}. \qquad (1)$$

We perform a complete eigenanalysis of $\mathcal{M}^{-1}(\omega)\mathcal{A}$ showing that the optimal value of $\omega$ can be put in connection with the largest positive eigenvalues of $\tilde{A} = P_A^{-1}A$ and $\tilde{S} = P_S^{-1}S$. Numerical results on geomechanical coupled consolidation problems of size up to $2 \times 10^6$ unknowns show that proper choice of $\omega$ based on a cheap estimation of spectral radius of $\tilde{A}$ and $\tilde{S}$ may lead to a 70% CPU time saving with respect to the *naive* MCP.

## References

[1] L. Bergamaschi, M. Ferronato and G. Gambolati, *Mixed constraint preconditioners for the solution to FE coupled consolidation equations*, J. Comp. Phys., 227 (2008), pp. 9885–9897.