Formal QD algorithm and Markov-Bernstein inequalities

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Let $L^2(\Omega;\mu_m)$, $m = 0, 1$, be Hilbert spaces of square integrable real functions on the open set $\Omega \subset \mathbb{R}$ for the positive Borel measures μ_m supported on Ω. The norm on this space is defined from the inner product

$$\| f \|_{L^2(\Omega;\mu_m)} = \left(\int_\Omega f(x)^2 d\mu_m\right)^{\frac{1}{2}}, \qquad \forall f \in L^2(\Omega;\mu_m), \quad m = 0, 1.$$

Let $\mathcal{P}_n$ be the vector space of real polynomials in one variable of degree at most equal to n. A Markov-Bernstein inequality corresponds to

$$||p'||_{L^2(\Omega;\mu_1)} \leq M_n ||p||_{L^2(\Omega;\mu_0)}, \quad \forall p \in \mathcal{P}_n,$$

where p' is the derivative of p. The smallest possible value of M_n is called the constant of Markov-Bernstein. It is well known that this best constant is linked with the smallest eigenvalue $\alpha_{1,n}$ of a $n \times n$ positive definite symmetric matrix : $M_n = \frac{1}{\sqrt{\alpha_{1,n}}}$. It is exceptional to obtain the exact value of M_n. So, it remains the solution to produce formal lower and upper bounds of $\alpha_{1,n}$ in order to give the asymptotic behavior of this eigenvalue.

Our aim is to do a review of the use of formal different versions of the *qd* algorithm in such a way that some sequences of upper bounds are given for this smallest eigenvalue $\alpha_{1,n}$ in the case of the classical measures (Laguerre-Sonin, Jacobi) and partially in the case of the generalized Gegenbauer measure.

A problem is still open : to use such a formal algorithm for finding an upper bound of the smallest eigenvalue of a generalized eigenvalue problem $Ax = \lambda Bx$, where (in our case) A and B are $n \times n$ positive definite symmetric matrices.