Extrapolation of operator moments, with applications to linear algebra problems

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Let A be a linear self-adjoint operator from H to H, where H is a real infinite dimensional Hilbert space with the inner product $(\cdot, \cdot)$. For positive powers of A, the Hilbert space H could be infinite dimensional, while, for negative powers it is always assumed to be a finite dimensional, and, in this case, A is also assumed to be invertible. Using the singular value decomposition for a compact linear self-adjoint operator A and its moments, we can define it's fractional powers by $A^\nu z = \sum_k \sigma_k^\nu (z, u_k) u_k$, and its fractional moments by $c_\nu(z) = (z, A^\nu z) = \sum_k \sigma_k^\nu \alpha_k^2(z)$, where $\alpha_k(z) = (z, u_k)$, for $\nu \in \mathbb{Q}$.

We will approximate $c_q(z)$ by interpolating or extrapolating, at the point $q \in \mathbb{Q}$, the $c_n(z)$'s for different values of the non-negative integer index n by a conveniently chosen function obtained by keeping only one or two terms in the preceding summations.

Estimates of the trace of A^q, for $q \in \mathbb{Q}$, and of the norm of the error when solving a system $Ax = f \in H$ will be given. For $q = -1$, other estimates of the trace of the inverse of a matrix could be found in [1, 2].

References

[1] C. Brezinski, P. Fika, M. Mitrouli, *Moments of a linear operator, with applications to the trace of the inverse of matrices and the solution of equations*, Numerical Linear Algebra with Applications, to appear.

[2] G. H. Golub, G. Meurant, *Matrices, Moments and Quadrature with Applications*, Princeton University Press, Princeton, 2010.