An application of the integrable discrete hungry Toda equation to the eigenvalue computation

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The qd algorithm [1], for computing eigenvalues, has a close relationship with the integrable discrete Toda equation. It is known that the discrete Toda equation is extended to the discrete hungry Toda (dhToda) equation through the study of the box and ball system [2].

In this talk, we propose a new matrix eigenvalue algorithm in terms of the dhToda equation. We first show that a time evolution from n to $n+1$ of the dhToda equation corresponds to a similarity transformation of a totally nonnegative (TN) matrix. Here the TN matrix is a matrix with all nonnegative minors. We next reveal that the the dhToda variable has a periodical asymptotic behavior. As n becomes sufficiently large, the implicit equilibrium points are related to the eigenvalues of the TN matrix. Based on this property, we design a new algorithm, named the dhToda algorithm, for eigenvalues of the TN matrix. Numerical examples show that the dhToda algorithm is with high relative accuracy. We also describe some other properties of the algorithm.

References

[1] H. Rutishauser, *Lectures on Numerical Mathematics*, Birkhäuser, Boston, 1990.

[2] T. Tokihiro, A. Nagai, J. Satsuma, *Proof of solitonical nature of box and ball systems by means of inverse ultra-discretization*, Inverse Problems, 15 (1999), pp. 1639–1662.