# Convergence of collocation method with delta functions for integral equations of first kind

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We consider integral equations of type $\int_\gamma K(x,y)u(y)dS_y = f(x)$, $x \in \Gamma$, where $\gamma$ and $\Gamma$ are some closed disjoint curves or surfaces. Equations of this type arise when solving boundary value problems of elliptic partial differential equations by interior source methods. These methods generate the solution of the differential equation as an integral over a contour or a surface outside the closure of the (usually exterior) domain. Typically $K$ has a singularity at $x = y$, and if $\Gamma$ and $\gamma$ are disjoint, the singularity is avoided. In fact, if $\Gamma$ and $\gamma$ are analytic, then the integral equation has an analytic kernel. Results about existence and uniqueness of the solution can often be obtained only in spaces of linear analytic functionals, and in general case, only density of the range of the integral operator can be proved.

We look for approximate solutions of the integral equation as linear combinations of Dirac's $\delta$-functions. For the corresponding collocation method, in case of analytic data the convergence is exponential in the number of variables. If the boundary is only piecewise smooth, the convergence rate deteriorates to algebraic. To get a more robust method, one can choose on $\Gamma$ more points than on $\gamma$, and solve the corresponding overdetermined system by least squares.

## References

[1] U. Kangro, *Convergence of Collocation Method with Delta Functions for Integral Equations of First Kind*, Integr. equ. oper. theory, **66**(2) (2010), pp. 265–282.