A new approach to control the global error of numerical methods for differential equations

G. Yu. Kulikov and R. Weiner
CEMAT (Centro de Matemática e Aplicações)
Instituto Superior Técnico, TU Lisbon
Av. Rovisco Pais, 1049-001 Lisboa, Portugal
gkulikov@math.ist.utl.pt

Recently, Kulikov presented the idea of double quasi-consistency, which facilitates global error estimation and control, considerably. More precisely, a local error control implemented in doubly quasi-consistent numerical methods plays a part of global error control at the same time. However, Kulikov studied only Nordsieck formulas and proved that there exists no doubly quasi-consistent scheme among those methods.

In this paper, we prove that the class of doubly quasi-consistent formulas is not empty and present the first example of such sort. This scheme belongs to the family of superconvergent explicit two-step peer methods constructed by Weiner et al. We present a sample of s-stage fixed-stepsize doubly quasi-consistent parallel explicit peer methods of order $s-1$ when $s=3$. Then, we discuss variable-stepsize explicit parallel peer methods grounded in the interpolation idea. Approximation, stability and convergence are studied in detail. In particular, we prove that some interpolation-type peer methods are stable on any variable mesh in practice. Double quasi-consistency is utilized to introduce an efficient global error estimation formula in the numerical methods under discussion. The main advantage of these new adaptive schemes is the capacity of producing numerical solutions for user-supplied accuracy conditions in automatic mode and almost at no extra cost. This means that a usual local error control mechanism monitors and regulates the global error at the same time because the true error of any doubly quasi-consistent numerical method is asymptotically equal to its local error. Numerical experiments support theoretical results of this paper and illustrate how the new global error control concept works in practice. We also conduct a comparison with explicit ODE solvers in MatLab.