A radial basis function based partition of unity method for solving PDE problems

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Radial basis function (RBF) based approximation methods are interesting in the context of PDE solving due to their ease of implementation, their potentially spectral convergence rates, and their flexibility with respect to geometry. However, a persistent problem has been the severe ill-conditioning of the systems of equations that typically need to be solved. This ill-conditioning is partly related to the size of the system, but even more so to the shape parameter of the RBFs. As the shape parameter is decreased, the RBFs become increasingly flat, leading to a nearly linearly dependent basis. However, the nearly flat limit in many cases provide the best approximation properties. The recently developed RBF-QR method [1] provides numerically stable evaluations for the small shape parameter range in up to three space dimension. With the conditioning obstacle removed, the focus can be turned to more general computational issues such as computational cost and memory requirements. Instead of using a global RBF method resulting in a dense linear system, we propose a partition of unity approach with local RBF approximants. The locality reduces both memory usage and computational cost compared with the global method. We show that the RBF-QR algorithm is a key to success and provide both theoretical results and numerical experiments showing spectral convergence with respect to the local problem size and algebraic convergence with respect to the partition size.

References

[1] B. Fornberg, E. Larsson, and N. Flyer *Stable computations with Gaussian radial basis functions*, SIAM J. Sci. Comput., 33 (2011), pp. 869-892.