Multivariate Prony's method

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In 1795, Gaspard Riche de Prony presented a method for interpolating a sum of exponential functions. Closely related to Padé approximation, Prony's method has found applications in the shape from moments problem, spectral estimation, and lately sparse sampling of digital signals with finite rate of innovation.

The interesting connection between Prony's method and error-correcting codes has led to the development of symbolic-numeric sparse polynomial interpolation, which has also exploited a reformulation of Prony's method as a generalized eigenvalue problem and a link to Rutishauser's qd-algorithm.

Recall that a meromorphic function is a function analytic everywhere except at a set of isolated points that are called the poles of the function. Rutishauser's qd-algorithm can determine the poles of a meromorphic function from its Taylor expansion. In the multivariate case such poles form a set of solutions of the associated multivariate polynomial equations. The interdependence between the Taylor expansion and poles becomes less obvious because there can be various ways to order the multivariate Taylor coefficients.

Recent progress in the multivariate qd-algorithm expands our understanding in associating the convergence of multivariate poles to the different orderings of Taylor coefficients, among which a special case has been implemented in developing numerical multivariate polynomial factorization. Resorting to the link from qd to Padé leads us to a multivariate Prony's method, which intricately involves higher-order tensors and their decompositions.