Fast direct solvers for elliptic PDEs

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That the linear systems of algebraic equations arising upon the discretization of elliptic PDEs can be solved very rapidly is well-known, and many successful iterative solvers with linear complexity have been constructed (multigrid, Krylov methods, etc). More recently, it has been demonstrated that it is also often possible to directly compute an approximate inverse (or LU/Cholesky factorization) to the coefficient matrix in linear or close to linear time. The inverse is computed in a data-sparse format that exploits internal matrix structure such as rank-deficiencies in the off-diagonal blocks.

The talk will focus on methods relying on the *Hierarchically Semi-Separable (HSS)* matrix format to efficiently represent the solution operator to the PDE. This format is less versatile than the more popular $\mathcal{H}$ and $\mathcal{H}^2$ matrix formats, but typically results in very high performance in terms of speed and accuracy when it can be made to work. For problems on 1D domains such as a boundary integral equation (BIE) on a domain in the plane, the adaptation of the HSS format is straight-forward, and problems in higher dimensions can be handled via recursive domain decomposition techniques that reduce the dimensionality of the domain on which the compressed operator acts.

The talk will describe numerical examples in both two and three dimensions. Variable coefficient problems are handled via accelerated nested dissection methods, while constant coefficient problems are solved via the corresponding boundary integral equation formulations.