We consider a quadrature method based on the alternate trapezoidal quadrature for the eigenvalue problem given by the Fredholm integral equation of second kind. The understanding of the spectral properties of this quadrature method is of interest to the numerical computation of the periodic Hilbert transform and related operators without resorting to the spectral method. We prove that, for some convolution-type integral kernels, such a quadrature method always yields eigenvalues with double multiplicity. This could help to reduce the computational cost of the quadrature method by exploring the decoupling of the resulting discrete eigenvalue problem.