Numerical methods for nonlinear two-parameter eigenvalue problems

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In a nonlinear two-parameter eigenvalue problem (NMEP) we are searching for a pair (λ, μ) and nonzero vectors x_1, x_2, such that

$$\begin{aligned} F_1(\lambda,\mu)x_1 &= 0, \\ F_2(\lambda,\mu)x_2 &= 0, \end{aligned}$$

where $F_i : \mathbb{C}^2 \to \mathbb{C}^{n_i \times n_i}$ is a nonlinear operator for $i = 1, 2$. In such case (λ, μ) is an eigenvalue and $x_1 \otimes x_2$ is the corresponding eigenvector. We assume that the problem is regular, i.e., $\det(F_i(\lambda,\mu)) \not\equiv 0$ for $i = 1, 2$.

NMEP can be viewed as a generalization of the nonlinear eigenvalue problem (NEP) as well as a generalization of the algebraic two-parameter eigenvalue problem (MEP) of the form

$$\begin{aligned} (A_1 + \lambda B_1 + \mu C_1)x_1 &= 0, \\ (A_2 + \lambda B_2 + \mu C_2)x_2 &= 0, \end{aligned}$$

where A_i, B_i, C_i are $n_i \times n_i$ complex matrices. We will show that many numerical methods and theoretical results for NEP and MEP can be generalized to NMEP.

An example of a NMEP is a quadratic two-parameter eigenvalue problem (QMEP) which appears in the study of linear time-delay systems for the single delay case.