Approximate greatest common divisors of Bernstein polynomials

N. Yang and **J. Winkler**
Department of Computer Science
The University of Sheffield
Regent Court, 211 Portobello, Sheffield S1 4DP, United Kingdom
j.winkler@dcs.shef.ac.uk

This paper considers the computation of the degree of an approximate greatest common divisor (AGCD) of two Bernstein basis polynomials $f(y)$ and $g(y)$. This computation is usually performed either by Euclid's algorithm, or by computing the singular value decomposition of the Sylvester resultant matrix $S(f, g)$, where $f = f(y)$ and $g = g(y)$, and determining the rank loss of this matrix. These methods give, however, poor results when $f(y)$ and $g(y)$ are corrupted by noise, which is the situation encountered in practical problems.

Two new methods for the computation of the degree of an AGCD of the inexact polynomials $f(y)$ and $g(y)$ are described, and computational results are presented when the greatest common divisor of the theoretically exact forms of $f(y)$ and $g(y)$ is of high degree. It is shown that the polynomials must be preprocessed by three operations before these methods are implemented. One of these preprocessing operations is the normalisation of the coefficients of $f(y)$ and $g(y)$, and it is shown that normalisation by the geometric means of their coefficients is superior to normalising by the 2-norms of their coefficients. The effect of the second preprocessing operation is the destruction of the Bernstein basis, that is, $f(y)$ and $g(y)$ are transformed to another basis. All computations are performed in the new basis, and the results obtained with this new basis are compared with the results obtained when the preprocessing operations are omitted, that is, the computations are performed in the Bernstein basis. It is shown that the inclusion of the preprocessing operations yields a considerable improvement in the computed results with respect to the results obtained when the computations are performed in the Bernstein basis, even when the theoretically exact forms of $f(y)$ and $g(y)$ have roots in the interval $[0, \dots, 1]$. A possible explanation for these improved results is considered.