Let $L^2(\Omega; \mu_m)$, $m = 0, 1$, be Hilbert spaces of square integrable real functions on the open set $\Omega \subset \mathbb{R}$ for the positive Borel measures $\mu_m$ supported on $\Omega$. The norm on this space is defined from the inner product

$$\| f \|_{L^2(\Omega; \mu_m)} = \left( \int_{\Omega} f(x)^2 d\mu_m \right)^{\frac{1}{2}}, \quad \forall f \in L^2(\Omega; \mu_m), \quad m = 0, 1.$$  

Let $\mathcal{P}_n$ be the vector space of real polynomials in one variable of degree at most equal to $n$. A Markov-Bernstein inequality corresponds to

$$\| p' \|_{L^2(\Omega; \mu_1)} \leq M_n \| p \|_{L^2(\Omega; \mu_0)}, \quad \forall p \in \mathcal{P}_n,$$

where $p'$ is the derivative of $p$. The smallest possible value of $M_n$ is called the constant of Markov-Bernstein. It is well known that this best constant is linked with the smallest eigenvalue $\alpha_{1,n}$ of a $n \times n$ positive definite symmetric matrix : $M_n = \frac{1}{\sqrt{\alpha_{1,n}}}$. It is exceptional to obtain the exact value of $M_n$. So, it remains the solution to produce formal lower and upper bounds of $\alpha_{1,n}$ in order to give the asymptotic behavior of this eigenvalue.

Our aim is to do a review of the use of formal different versions of the qd algorithm in such a way that some sequences of upper bounds are given for this smallest eigenvalue $\alpha_{1,n}$ in the case of the classical measures (Laguerre-Sonin, Jacobi) and partially in the case of the generalized Gegenbauer measure.

A problem is still open : to use such a formal algorithm for finding an upper bound of the smallest eigenvalue of a generalized eigenvalue problem $Ax = \lambda Bx$, where (in our case) $A$ and $B$ are $n \times n$ positive definite symmetric matrices.