Universal analytic properties of noise

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We propose a new method in the spectral analysis of noisy time-series data for damped oscillators. From the Jacobi three terms recursive relation for the denominators of the Padé approximations built on the well-known Z-transform of an infinite time series, we build a Hilbert space operator, a J-operator, where each bound state (inside the unit circle in the complex plane) is simply associated with one damped oscillator while the essential spectrum of the J-operator, which lies on the unit circle itself, is shown to represent the noise.

Signal and noise are thus clearly separated in the complex plane. For a finite time series of length 2N, the J-operator is replaced by a finite order J-matrix J_N, having N eigenvalues which are time reversal covariant. Different classes of input noise, such as blank (white and uniform), gaussian and pink, are discussed in detail, the J-matrix formalism allowing us to efficiently calculate hundreds of poles of the Z-transform. Evidence of a universal behavior in the final statistical distribution of the associated poles and zeros of the Z-transform is shown. In particular, the poles and zeros tend, when the length of the time series goes to infinity, to a uniform angular distribution on the unit circle.

Therefore at finite order, the roots of unity in the complex plane *appear to be noise attractors.* We show that the Z-transform presents the exceptional feature of allowing *lossless undersampling* and how to make use of this property. A few basic examples are given to suggest the power of the proposed method.