# Shell models: old and new

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Intrinsic methods in elasticity have been introduced in a landmark series of papers by Wei-Zhang Chien in 1944. During the last two decades, Professor Wojciech Pietraszkiewicz and his group have achieved major advances in their analysis from the mechanical and engineering viewpoints, as well as in their actual numerical implementation. However, it was only in 2005 that their mathematical analysis began to be carried out in earnest, first for three-dimensional elasticity and more recently for elastic shells, by the author and his group. This presentation, which is intended for a general audience, will briefly review and discuss various problems as yet unresolved when this approach is applied to shell structures. In the classical approach, the main mathematical challenge is to establish that the associated energy has a minimizer. In the linear case, this is achieved through a "Korn inequality on a surface", which guarantees the positive-definiteness of the associated energy. In the nonlinear case, the problem remains basically open for Koiter's model, which is one of the most commonly used nonlinear models in numerical simulations. In the intrinsic approach, the main challenges lie not only in the mathematical analysis, but in effect in the modeling itself. Since the new unknowns are the change of metric and change of curvature tensor fields (instead of the displacement field in the classical approach), the Gauss and Codazzi-Mainardi compatibility equations conditions (or other equivalent equations) must be satisfied by these new unknowns, in order that they indeed correspond to a displacement of the middle surface of the shell. Another challenge is to adequately express boundary conditions in terms of these new unknowns. We will briefly review the existence theorems that has been recently obtained in the linear case. Besides, we will give in particular an explicit form of the compatibility conditions, as well as an explicit "Cesaro-Volterra integral formula on a surface" for reconstructing a displacement field from the knowledge of these new unknowns.