Rational Lanczos methods for the approximation of matrix functions

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The need to evaluate expressions of the form $f(A)v$ and $v^T f(A)v$, where A is a large, sparse or structured, invertible symmetric matrix, v is a vector, and f is a nonlinear function, arises in many applications. The extended Krylov subspace method can be an attractive scheme for computing approximations of such expressions. This method projects the approximation problem onto an extended Krylov subspace

$$\mathcal{K}^{\ell,m}(A,v) = \mathrm{span}\{A^{-\ell+1}v, \ldots, A^{-1}v, v, Av, \ldots, A^{m-1}v\}$$

of fairly small dimension, and then solves the small projected approximation problems so obtained. Orthonormal bases for extended Krylov subspaces can be generated with short recursion formulas, which can be derived using properties of Laurent polynomials. We will discuss the structure of the projections of the matrices A and A^{-1} onto $\mathcal{K}^{\ell,m}(A,v)$. This structure helps us derive efficient algorithms and relate projections of $v^T f(A)v$ to rational Gauss-type quadrature rules. The talk presents joint work with C. Jagels.