Interlacing properties of generalized Laguerre zeros and some applications

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Let $w(x) = e^{-x^\beta} x^\alpha$, $\alpha > -1$, $\beta > \frac{1}{2}$ be a Generalized Laguerre weight, and denote by $\{p_m(w)\}$ the corresponding sequence of orthonormal polynomials. Setting $\bar{w}(x) = xw(x)$, let $\{p_m(\bar{w})\}$ the sequence of orthonormal polynomials corresponding to $\bar{w}$. We prove that the polynomial $Q_{2m+1} = p_{m+1}(w)p_m(\bar{w})$ has simple zeros and that they are also well distributed in some sense.

In view of this property we propose two different applications: the *extended interpolation polynomial* $L_{2m+2}(w, \bar{w}, f)$, defined as the Lagrange polynomial interpolating a given function f at the zeros of Q_{2m+1} and on additional knots, estimating the Lebesgue constants in some weighted spaces. Moreover, we propose a method to approximate the Hilbert transform on the real positive semiaxis by a suitable Lagrange interpolating polynomial.