Computing Fekete and Lebesgue points: simplex, square, disk

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The main purpose of our work is to provide Fekete and Lebesgue points on three basic bidimensional compact sets, the simplex, the square, and the disk, by solving numerically the corresponding large-scale nonlinear optimization problems up to degree $n = 18$. Our results reach and often improve those previously known [1], [3]. In the case of the simplex, due to their relevance in developing spectral and high-order methods for PDEs [2] we have also computed interpolation sets that have an assigned distribution on the sides (Legendre-Gauss-Lobatto side nodes), which appear to be better than those previously known. Concerning the square, besides Fekete and Lebesgue points, we have computed some new sets that generalize the Padua points and improve their already good quality. Very little seems to be known about Fekete and Lebesgue points for the disk, and we hope that our computational results could put some insight into this topic.

References

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