Fourier series and acceleration methods

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For a function f which is integrable on $[-\pi, \pi]$, we consider the partial sums of its Fourier series

$$S_n f(x) := \sum_{k=-n}^{n} \hat{f}(k) e^{ikx}.$$

We transform the sequence of partial sums using the δ^2 and the Lubkin transforms to obtain respectively the sequences of functions

$$T_n f(x) := \frac{S_{n+1} f(x) S_{n-1} f(x) - (S_n f(x))^2}{S_{n+1} f(x) + S_{n-1} f(x) - 2 S_n f(x)}$$

(where we set $T_n f(x) = S_n f(x)$ if the denominator of the fraction is zero) and

$$S_n^* f(x) := S_n f(x) + \frac{(S_{n+1} f(x) - S_n f(x))(1 - \rho_{n+1} f(x))}{1 - 2\rho_{n+1} f(x) + \rho_n f(x) \rho_{n+1} f(x)}.$$

where $\rho_n f(x) = (S_{n+1} f(x) - S_n f(x))/(S_n f(x) - S_{n-1} f(x))$.

Both of these transforms fail to accelerate convergence in general: For functions which are smooth, except for a single jump discontinuity, both transforms diverge on a dense set. We also construct Hölder continuous functions, analytic on the interior of the unit disk, for which the transformed sequences fail to converge at every point. We discuss iterations of these transforms and the epsilon algorithm.