Extensions of Padé-type approximants

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A Padé approximant $[l/m]_f(z)$ to a function $f(z)$ is the ratio of two polynomials $P^{[l/m]}(z) = p_0 + p_1 z + \cdots + p_l z^l$ and $Q^{[l/m]}(z) = 1 + q_1 z + \cdots + q_m z^m$. The polynomial coefficients can be determined via the *accuracy-through-order* relationship $Q^{[l/m]}(z) f(z) - P^{[l/m]}(z) = \mathrm{O}(z^{l+m+1})$ as $z \to 0$.

It is normally highly advantageous that for the computation of $[l/m]_f(z)$ only the numerical values of the partial sums $f_n(z) = \sum_{\nu=0}^{n} \gamma_\nu z^\nu$ with $0 \le n \le l + m$ of the (formal) power series for $f(z)$ have to be known. No additional information is necessary. But this also means that there is no obvious way of incorporating additional information about f or the index dependence of the partial sums $f_n(z)$ into the transformation process, although such an information may be available.

As a remedy, Brezinski [J. Approx. Theory, 25 (1979), pp. 295 – 317] proposed so-called *Padé-type* approximants $(l/m)_f(z) = \mathcal{U}^{(l/m)}(z)/\mathcal{V}^{(l/m)}(z)$, which are also ratios of two polynomials. But now, it is assumed that the coefficients of the the denominator polynomial are known. Thus, only the coefficients of the numerator polynomial have to be determined via the modified *accuracy-through-order* relationship $\mathcal{V}^{(l/m)}(z) f(z) - \mathcal{U}^{(l/m)}(z) = \mathrm{O}(z^{l+1})$ as $z \to \infty$.

The zero's of the denominator polynomial correspond to the poles of a Padé-type approximant. Unfortunately, there are not too many non-trivial functions whose pole structure is fully understood. Accordingly, indirect approaches for the choice of the denominator polynomials of Padé-type approximant have to be pursued.

This talk first discusses certain Levin-type transformations, which are known to be very powerful convergence acceleration and summation techniques and which are actually special Padé-type approximants [E. J. Weniger, J. Math. Phys., 45 (2004), pp. 1209 – 1246, Section VI]. Then, some examples from special function theory are discussed which show how denominator polynomials can be chosen by utilizing knowledge about the location of the cuts of the function.