Conjugate gradient iteration under white noise

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Conjugate gradient iteration for ill-posed problems in Hilbert space, say given by an operator equation

$$y^{\delta} = Ax + \delta\xi,$$

where $A\colon X \to Y$ acts between Hilbert spaces, is well known to yield regularization if it is stopped according to the discrepancy principle. The seminal work in this direction is [2]. This presumes that the data y^{δ} are noisy, but the noise is bounded in the original norm of Y. If, instead, we assume that the noise ξ is white noise, i.e., it is centered and has identity covariance operator, then the data y^{δ} will not belong to Y (a.s.), and hence the discrepancy is not well defined.

Based on previous work [1] we propose a modified discrepancy principle and we show that order optimal regularization can be achieved. These modifications concern both, the norm in which the discrepancy is evaluated and an emergency stop, which guarantees that the iteration stops even if the data at hand behave badly.

This modified discrepancy principle also works for linear regularization of statistical ill-posed problems.

This is joint work with G. Blanchard, Potsdam University.

References

[1] Gilles Blanchard and Peter Mathé. Conjugate gradient regularization under general smoothness and noise assumptions. *J. Inv. Ill-Posed Problems*, 18 (2010), pp. 701–726.

[2] A. S. Nemirovskiĭ, *Regularizing properties of the conjugate gradient method in ill-posed problems.* Zh. Vychisl. Mat. i Mat. Fiz. 26 (1986), pp. 332–347.