Exact solutions to the focusing discrete nonlinear Schrödinger equation

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In this talk, by using the Inverse Scattering Transform (IST), we derive explicit solutions of the discrete nonlinear Schrödinger equation (IDNLS):

$$i\frac{d}{d\tau}\mathbf{u}_n = \mathbf{u}_{n+1} - 2\mathbf{u}_n + \mathbf{u}_{n-1} + \mathbf{u}_{n+1}\mathbf{u}_n^\dagger\mathbf{u}_n + \mathbf{u}_n\mathbf{u}_n^\dagger\mathbf{u}_{n-1}\,, \tag{1}$$

where n is an integer and $\mathbf{u}_n$ is an $N \times M$ matrix function depending on time τ. More precisely, the IST associates (1) to the following discrete Zakharov-Shabat system,

$$\mathbf{v}_{n+1} = \begin{pmatrix} zI_N & \mathbf{u}_n \\ \mathbf{u}_n^\dagger & z^{-1}I_M \end{pmatrix} \mathbf{v}_n \tag{2}$$

where z is the (complex) spectral parameter.

We get the explicit solutions for (1) announced above by applying the Marchenko method to solve the inverse problem associated with (2). In fact, representing the kernel of the Marchenko equation as

$$CA^{-(n+j+1)}e^{i\tau(A-A^{-1})^2}B,$$

where (A, B, C) is a matrix triplet such that the $p \times p$ matrix A has only eigenvalues of modulus larger than one, while B and C have sizes, respectively, $p \times N$ and $M \times p$, the Marchenko equation can be solved explicitly by separation of variables. The class of solutions obtained contains the N-soliton and the breather solutions as special cases, as well as the so-called multipole soliton solutions.