Commuting vector fields and integrable PDEs of hydrodynamic type

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Using a recently developed Inverse Spectral Transform for commuting multidimensional vector fields [1], we have been able to solve the Cauchy problem for physically relevant PDEs, like the heavenly equation of Plebansky [1], relevant in General Relativity, the dispersionless Kadomtsev - Petviashvili (dKP) equation [2], describing the propagation of weakly nonlinear and quasi one dimensional waves in the absence of dispersion and dissipation, and the 2D dispersionless Toda (2DDT) equation [3], describing integrable Einstein - Weyl metrics and ideal Hele - Shaw flows. In addition, the associated nonlinear Riemann - Hilbert inverse problem has turned out to be a powerful tool to study the longtime behavior of solutions, to construct classes of exact implicit solutions and to study in great detail the gradient catastrophe of multidimensional waves, like in the case of the dKP and 2DDT equations.

References

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