Novel formulation of discrete integrable nonlinear Schrödinger equations

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Discretizations of matrix nonlinear Schrödinger equations have the problem that the natural finite difference discretization of the matrix NLS equation leads to a nonlinear equation whose integrability is not obvious. This has led to the development of an inverse scattering transform scheme where the (focusing) discrete NLS solution $\boldsymbol{u}_n(t)$ is required to let $\boldsymbol{u}_n(t)\boldsymbol{u}_n(t)^\dagger$ and $\boldsymbol{u}_n(t)^\dagger\boldsymbol{u}_n(t)$ be nonzero multiples of the identity matrix, thus preventing a proper discretization of the Manakov system.

In this talk we explore various remedies. One is to discretize every single step in the IST, but in this case the nonlinear evolution problem might be difficult to formulate (although integrability is guaranteed). The other option is to apply central differencing (and not one-sided differencing as Ablowitz-Ladik did) in the matrix Zakharov-Shabat system, develop the direct and inverse scattering theory of the resulting system

$$iJ\frac{u_{n+1}-u_{n-1}}{2h} = [\lambda I_{N+M} + U_n]u_n,$$

stick in time factors, and apply the usual matrix triplet method to develop explicit discrete NLS solutions. Again we have to hope knowing the nonlinear evolution system. Here we attempt finding suitable Lax pairs.