Asymptotics for Christoffel functions based on orthogonal rational functions

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Suppose the rational functions $\{\varphi_j\}$, with poles in $\{\alpha_1, \ldots, \alpha_j\} \subset (\mathbb{C} \cup \{\infty\}) \backslash [-1, 1]$, form an orthonormal system with respect to a positive bounded Borel measure μ on $I := [-1, 1]$, satisfying the Erdős-Turán condition $\mu' > 0$ a.e. on I, and let the associated Christoffel functions be given by $\lambda_n(x) = [\sum_{j=0}^{n-1} |\varphi_j(x)|^2]^{-1}$. Assuming the sequence $\{n\lambda_n(x)\}_{n>0}$ converges for certain $x \in I$, and the poles are all real and bounded away from I, in [2, Appendix A.2] the author obtained an expression for the limit function $k(x) = \lim_{n\to\infty} n\lambda_n(x)$. The actual convergence, however, has only been proved for the special case of the Chebyshev weight functions $\frac{d\mu(t)}{dt} = (1+t)^a(1-t)^b$, where $a, b \in \{\pm\frac{1}{2}\}$, and for every $x \in I$ in [2, Chapter 9.7]. In this contribution we will prove convergence for arbitrary complex poles bounded away from I, and weight functions of the form $\frac{d\mu(t)}{dt} = g(t) \prod_{i=1}^{k} |t - t_i|^{\nu_i}$, where $-1 \leqslant t_1 < \ldots < t_i < \ldots < t_k \leqslant 1$, $\nu_i > -1$, $0 < C_1 \leqslant g(t) \leqslant C_2 < \infty$ for every $t \in I$, and $g(t)$ is continuous in a neighbourhood of $x \in I$.

References

[1] A. Bultheel, P. González-Vera, E. Hendriksen, and O. Njåstad, *"Orthogonal Rational Functions"*, vol. 5 of Cambridge Monographs on Applied and Computational Mathematics, Cambridge U.P., Cambridge, 1999. (407 pages).

[2] J. Van Deun, *Orthogonal Rational Functions: Asymptotic Behaviour and Computational Aspects*, PhD Thesis, K.U.Leuven, Dept. of Computer Science, May 2004.