Properties and applications of constrained dual Bernstein polynomials

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Constrained dual Bernstein polynomials $D_i^{(n,k,l)}(x;\alpha,\beta) \in \Pi_n^{(k,l)}$ $(i = k, k+1, \ldots, n-l;\ 0 \le k+l \le n)$ are defined so that

$$\int_0^1 (1-x)^\alpha x^\beta D_i^{(n,k,l)}(x;\alpha,\beta) B_j^n(x)dx = \delta_{ij} \qquad (k \le i, j \le n-l),$$

where $\alpha, \beta > -1$, and

$$B_j^n(x) := \binom{n}{j} x^j (1-x)^{n-j} \qquad (j = 0, 1, \ldots, n)$$

are basis Bernstein polynomials. Here $\Pi_n^{(k,l)}$ denotes the space of all polynomials of degree n, whose derivatives of order $\le k-1$ at $t=0$, as well as derivatives of order $\le l-1$ at $t=1$, vanish.

Polynomials $D_i^{(n,k,l)}(x;\alpha,\beta)$ are closely related to some families of orthogonal polynomials, namely shifted Jacobi and Hahn polynomials. We show many properties of constrained dual Bernstein polynomials, as well as quantities related to them. Using these results, we propose efficient algorithms of solving some approximation problems which appear in computer aided geometric design.