Symbolic-numeric integration, multivariate orthogonality and Padé approximation

A. Cuyt
Universiteit Antwerpen,
Department of Mathematics and Computer Science
Middelheimlaan 1, B-2020 Antwerpen, Belgium
annie.cuyt@ua.ac.be

Moving from one to more dimensions with polynomial-based numerical techniques leaves room for a lot of different approaches and choices. We focus here on rational approximation, orthogonal polynomials and integration rules, three very related concepts.

In one variable there is a close connection between orthogonal polynomials, Gaussian quadrature rules and Padé approximation. An m-point Gaussian quadrature formula for the integral

$$I(z) = \int_{-1}^{1} \frac{1}{1-tz}\, dt$$

can be viewed as the $[m-1/m]$ Padé approximant for the function

$$f(z) = \sum_{i=0}^{\infty} \left(\int_{-1}^{1} t^i\, dt \right) z^i$$

where the nodes and weights of the Gaussian quadrature formula are obtained from the orthogonal polynomial $V_m(z)$ satisfying

$$\int_{-1}^{1} z^i V_m(z)\, dz = 0, \qquad i = 0, \ldots, m-1.$$

We show that this close connection can be preserved in several variables when starting from spherical orthogonal polynomials. We obtain Gaussian cubature rules with symbolic nodes and numeric weights which can be used to integrate parameterized families of functions. The spherical orthogonal polynomials are also related to the homogeneous Padé approximants introduced a few decades ago.