Smoothing the Gibbs phenomenon using Padé-Hermite approximants

B. Beckermann, V. Kaliagyn, **A. Matos**, and F. Wielonsky
UFR de Mathématiques
Université de Lille 1
59655 Villeneuve d'Ascq Cedex, France
Ana.Matos@univ-lille1.fr

The aim of this talk is to propose a method to reduce the Gibbs phenomenon exhibited by the partial Fourier sums of a periodic function f, defined on $[-\pi, \pi]$, discontinuous at 0. Let g_2 denote the series such that $f(t) = \Re(g_2(e^{it}))$. Then, the goal is to approach g_2 on the unit circle (and more precisely its real part). It is typical that the singularity of the function f, located at 0 say, corresponds to a logarithmic singularity for g_2, then located at 1, and that this function g_2 is analytic in the complex plane, with a branch cut that can be taken as the interval $[1, \infty)$. Defining $g_1(z) = \log(1-z)$, we may consider the problem of determining polynomials p_0, p_1, p_2 such that

$$p_0(z) + p_1(z)g_1(z) + p_2(z)g_2(z) = O(z^{n_0+n_1+n_2+2}) \quad (z \to 0)$$

where n_j denotes the degree of p_j, $j = 0, 1, 2$. We can then propose the *Hermite-Padé approximant*

$$\Pi_{\vec{n}}(z) = -\frac{p_0(z) + p_1(z)g_1(z)}{p_2(z)}, \tag{1}$$

to approximate g_2.

We obtain rates of convergence of sequences of Hermite–Padé approximants for a class of functions known as *Nikishin systems*. Our theoretical findings and numerical experiments confirm that particular sequences of Hermite-Padé approximants (diagonal and row sequences, as well as linear HP approximants) are more efficient than the more elementary Padé approximants, particularly around the discontinuity of the goal function f.