Reliable reduced basis method for efficient geometrical parametrizations

A. Quarteroni, T. Lassila, A. Manzoni, G. Rozza
EPFL, MATHICSE CMCS
alfio.quarteroni@epfl.ch

In the last decades optimal control and shape optimization problems have gained an increasing importance in many engineering fields and especially in structural mechanics and in thermo-fluid dynamics. The problems we consider involve the study of a system modelled by parametric PDEs and the evaluation of functionals depending on the field variables, such as velocity, pressure, drag forces, temperature, energy, wall shear stress or vorticity. Especially in the field of shape optimization and parameter estimation, where the recursive evaluation of the solution is required for many possible configurations, the computational costs can easily become unacceptably high. Nevertheless, the evaluation of an input/output relationship of the system plays a central role: a set of input parameters identifies a particular configuration of the system and they may represent design and/or geometrical variables, while the outputs may be expressed as functionals of the field variables associated with PDEs. The rapid and reliable evaluation of many input/output relationships typically would require great computational expense, and therefore strategies to reduce the computational time and effort are needed. Among model order reduction strategies, reduced basis method represents a promising tool for the simulation of flow in parametrized geometries, for shape optimization or sensitivity analysis. An implementation of the reduced basis method is presented by considering different shape or domain parametrizations: from simple affine and non-affine maps, to more flexible techniques, such as free-form deformations or radial basis functions. In order to develop efficient numerical schemes for inverse problems related with shape variation such as shape optimization, fluid-structure interaction, shape analysis through parameter identification, we combine a suitable low-dimensional parametrization of the geometry (yielding a geometrical complexity reduction) with reduced basis methods (yielding a reduction of computational complexity). The analysis will focus on the general properties and performance of the reduced basis method: several examples will highlight its special suitability for the analysis of flows in parametrized geometries with a special interest in cardiovascular problems.