Recovery of eigenvectors of matrix polynomials from generalized Fiedler linearizations

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A standard way to solve polynomial eigenvalue problems $P(\lambda)x = 0$ is to convert the matrix polynomial $P(\lambda)$ into a matrix pencil that preserves its elementary divisors and, therefore, its eigenvalues. This process is known as linearization and is not unique, since there are infinitely many linearizations with widely varying properties associated with $P(\lambda)$. This freedom has motivated the recent development and analysis of new classes of linearizations that generalize the classical first and second Frobenius companion forms, with the goals of finding linearizations that retain whatever structures that $P(\lambda)$ might possess and/or of improving numerical properties, as conditioning or backward errors, with respect the companion forms. In this context, an important new class of linearizations is what we name generalized Fiedler linearizations, introduced in 2004 by Antoniou and Vologiannidis as an extension of certain linearizations introduced previously by Fiedler for scalar polynomials. On the other hand, the mere definition of linearization does not imply the existence of simple relationships between the eigenvectors, minimal indices, and minimal bases of $P(\lambda)$ and those of the linearization. So, given a class of linearizations, to provide easy recovery procedures for eigenvectors, minimal indices, and minimal bases of $P(\lambda)$ from those of the linearizations is essential for the usefulness of this class. In this paper we develop such recovery procedures for generalized Fiedler linearizations and pay special attention to structure preserving linearizations inside this class.