How to evaluate the historical and epistemological role of practical computing methods for the fundamental theorem of algebra

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By the end of the 18th century, many proofs had already been proposed for the fundamental theorem of algebra, by Euler, by d'Alembert (around 1750), by Lagrange, de Foncenex, and Laplace (around 1795). All such proofs, except d'Alembert's one, required the existence of "imaginary" quantities, and the proof was to reduce such quantities to complex numbers. In 1799, Gauss in his Dissertation thesis, proposed a proof inspired by d'Alembert, but requiring a rather difficult result on algebraic curves entering a bounded closed subset of the topological plane, more difficult in fact that the theorem to be proved. All such algebraic proofs were indirect ones, based as if it was obvious on the intermediate value theorem, which was then conceived as a computing result. In many papers of the time, there was a sort of antinomy between computing and what we may call commutative algebra. Using an idea of Legendre in number theory (in fact the maximum principle idea for holomorphic functions), Argand who was not known in the main mathematical circles in Paris, proposed in 1806 a direct and elementary proof, as well as his famous representation of imaginary quantities (this time such quantities indeed were complex numbers). There is a failure in the proof, and later Cauchy, using Argand's ideas, went back to an indirect proof. But the idea was maintained that there was something in Argand's proof, which was coming from computing methods and had an algorithmic flavour. An elementary but rather sophisticated proof by Kneser settled the situation in our in the middle of the 20th century. My aim is to discuss, on this concrete example, a sort of mentality of mathematicians concerning practical methods as opposed to theoretical ones.