

EULER, LAGRANGE, RITZ, GALERKIN, COURANT, CLOUGH:  
ON THE ROAD TO THE FINITE ELEMENT METHOD

**M. J. Gander**, and G. Wanner  
Department of Mathematics, University of Geneva  
CP 64, 1211 Geneva 4, Switzerland  
`martin.gander@unige.ch`

The finite element method has become indispensable for the numerical simulation of partial differential equations. But where does this method come from? How was it invented? I will show in my talk how everything started with Euler and Lagrange, and their discrete and continuous formulations of variational problems, which led to the highway of variational calculus. We will then see the fundamental contributions of the Swiss physicist and mathematician Walther Ritz in detail, and his method to compute Chladni figures. The development went further on a detour to Russia, to Timoshenko, Bubnov and Galerkin, who immediately realized the importance of Ritz' method, and used it to solve hard problems in science and engineering. The western world in contrast was at that time more interested in existence and uniqueness proofs around Hilbert and Courant. The value of Ritz' invention was only recognized much later by Courant, who presented the first finite element calculation we were able to find in an address to the AMS. The name Finite Element Method was finally coined by Ray Clough and collaborators at Boeing. The mathematical development of the finite element method was then however just to begin.