A family of P-stable linear multistep methods for second order IVPs

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In the last years the study of the numerical solution of initial value problems (IVPs) for second order ODEs of special type given by

$$y''(x) = f(x, y)\,, \quad y(x_0) = y_0\,, \quad y'(x_0) = y'_0, \qquad x \in [x_0, X]\,, \qquad (1)$$

having periodic and oscillatory solution, has attracted the interest of many authors. When the problem to be solved is *stiff,* namely when its solution is a combination of components with dominant short frequencies and components with large frequencies and small amplitudes, the use of schemes satisfying "good" stability properties is mandatory. Following the idea of Dahlquist, a rigorous definition of them was given by Lambert and Watson in [2] for Linear Multistep Methods (LMMs) which solve (1) in its original formulation. In such paper they established that the order of a P-stable LMM, used as Initial Value Method (IVM), cannot exceed two.
In this talk, it will be shown that the use of LMMs as Boundary Value Methods (BVMs) [1] may be successful in overcoming the barrier of Lambert and Watson. In particular, a family of P-stable BVMs, called PGSCMs and obtained as a generalization of the Störmer-Cowell schemes, will be presented.

References

[1] L. Brugnano and D. Trigiante, *Solving ODEs by Linear Multistep Initial and Boundary Value Methods*, Gordon & Breach, Amsterdam, 1998.

[2] J. D. Lambert and I. A. Watson, *Symmetric multistep methods for periodic initial value problems*, J. Inst. Math. Appl., 18 (1976), pp.189–202.