Scalable AMG preconditioners for PDE-constrained optimization problems

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PDE-constrained optimization problems arise in many applications, such as optimal control, shape design, and parameter estimation. Their size and complexity demands for efficient numerical methods able to exploit high-performance computing resources. In this context, motivations for using multigrid methods [1] include their optimal convergence rates as well as the possibility of developing scalable implementations. We focus on parallel algebraic multigrid (AMG) preconditioners for the solution, through Krylov methods, of large-scale linear systems resulting from the discretization of the optimality conditions for distributed elliptic optimal control problems. We present AMG preconditioners based on Schwarz methods and a modification of the smoothed aggregation coarsening technique. This modification exploits the block structure of the matrix that results from ordering the unknowns so that the degrees of freedom corresponding to the same node of the discretization grid are consecutive. The aggregation and the prolongation and restriction operators are built by using only the entries of the blocks corresponding to the PDE constraint. This approach has been implemented within the framework of the parallel preconditioning package MLD2P4 [2]. Numerical experiments show the effectiveness of the preconditioners and their parallel efficiency.

References

[1] A. Borzì and V. Schulz, *Multigrid methods for PDE optimization*, SIAM Review, 51, 2 (2009), pp. 361–395.

[2] P. D'Ambra, D. di Serafino, and S. Filippone, *MLD2P4: A Package of Parallel Algebraic Multilevel Domain Decomposition Preconditioners in Fortran 95*, ACM Trans. Math. Software, 37, 3 (2010), article n. 30.