# Appell polynomials of second kind and related interpolation problem

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Classical Appell polynomials are defined by differential equation $A_n' = nA_{n-1}$, $n \in \mathbb{N}$, where $n \in \mathbb{N}$ is the degree of polynomials $A_n(x)$. Now we define Appell polynomials of second kind the polynomials given by

$$\Delta \mathcal{A}_n = n\mathcal{A}_{n-1}, \quad n \in \mathbb{N} \tag{1}$$

where $\Delta$ is the classical difference operator: $\Delta f(x) = f(x+1) - f(x)$.
Well known examples are Bernoulli polynomials of second kind and Boole polynomials.
Let $L$ be a linear functional on the space of real function defined in $[0, 1]$. We look for a polynomial $P_n(f, x)$ of degree $n$ such that

$$L\left(\Delta^k P_n\right) = L\left(\Delta^k f\right), \quad k = 0, ..., n. \tag{2}$$

We prove that this interpolation problem has the unique solution

$$P_n(f, x) = \sum_{k=0}^{n} \frac{L\left(\Delta^k f\right)}{k!} \mathcal{A}_n^L \tag{3}$$

where $\mathcal{A}_n^L$ is the class of Appell polynomials of second kind related to $L$.

## References

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