On the good pivots of Hadamard matrices and related issues

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We introduce for the first time the notion of *good pivot patterns* as follows. A pivot pattern $\{p_1, p_2, \ldots, p_n\}$ appearing after application of Gaussian Elimination (GE) with complete pivoting on a matrix of order n is called *good*, if its pivots satisfy $p_i p_{n-i+1} = n, \quad i = 1, \ldots, n$. Clearly, good pivot patterns are of the form $\left\{p_1, p_2, \ldots, p_{\frac{n}{2}}, \frac{n}{p_{\frac{n}{2}}}, \ldots, \frac{n}{p_2}, \frac{n}{p_1}\right\}$.

It is important to specify the possible existence of good pivot patterns appearing after application of GE on Completely Pivoted (CP) Hadamard matrices of various orders [2]. The appearance of this property confirms Cryer's conjecture [1] for all Hadamard matrices possessing good pivots and for those belonging to the same H-equivalence class, namely that their growth factor is equal to their order.

We shall prove that for every pivot p_k, $k = 2, \ldots, n$, of a CP Hadamard matrix H of order n it holds $p_k > 1$. Based on this fact, we take a small step towards the equality portion of Cryer's conjecture proving that if the pivots $\{p_1, p_2, \ldots, p_n\}$ of a CP Hadamard matrix H of order n are good, then its growth is equal to its order.

Hadamard matrices are the only matrices known so far that lead to good pivot patterns.

References

[1] C. W. Cryer, *Pivot size in Gaussian elimination*, Numer. Math., 12 (1968), pp. 335–345.

[2] C. Kravvaritis and M. Mitrouli, *The growth factor of a Hadamard matrix of order 16 is 16*, Numer. Linear Algebra Appl., 16 (2009), pp. 715-743.